SonTek Technical Notes

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Argonaut Internal Flow Theoretical Modeling
Firmware Ver. 9.0

Overview

Internal flow calculations are used by the Argonaut-SL and SW for real time flow data.

- Flow is based on velocity multiplied by cross sectional area.
- Water level is measured using a vertical acoustic beam; cross sectional area is calculated based on user-supplied channel dimensions.
- The firmware supports three channel types: irregular (i.e. natural stream, defined by up to 20 survey points), trapezoidal (i.e. regular concrete lined channel), and pipe. For flow calculations, irregular and trapezoidal channels are treated the same (open channels).
- Velocity is based on the Argonaut measured velocity, which is modified either by a user-supplied index calibration or by a theoretical model of how measured velocity compares with the mean velocity throughout the channel.
- This document describes the theoretical models used to relate Argonaut measured velocity to mean channel velocity.
- An empirically determined index velocity relation will be more accurate than a theoretical approach. However, the theoretical approach may provide reasonably accurate results without the independent measurements needed to develop the index velocity relation.

Theoretical Approach

The approach largely comes from Chen (1991), starting with the power velocity distribution.

Equation 1 $$\frac{u}{u*} = a\left(\frac{y}{y'}\right)^m$$

Where

u	= velocity at some location y within the channel
u*	= boundary shear velocity
a	= constant
m	= constant
y	= normal range to the boundary
y'	= reference distance for boundary shear velocity

A more useful formula from our point of view looks at the relation between velocity at a point within the channel to the mean velocity throughout this channel. This is obtained by integrating the above relation over the geometry of the channel. For an example, consider a rectangular channel (constant depth). Ignorning the effects of the channel walls, we integrate the vertical profile to get the mean velocity in the channel.

Equation 2 $$V = \frac{\int_0^h u\,dy}{h} = \frac{\int_0^h au*\left(\frac{y}{y'}\right)^m dy}{h}$$

This can be reduced to:

Equation 3 $$\frac{V}{u*} = \frac{a}{m+1}\left(\frac{h}{y'}\right)^m$$

Where:

V = mean velocity in the channel
h = channel depth

Combining Equation 1 and Equation 3 gives an expression for the mean velocity throughout the channel based on the velocity measured at a given depth.

Equation 4 $$\frac{V}{u} = \frac{1}{m+1}\left(\frac{h}{y}\right)^m$$

We consider the ratio (V/u) to be a velocity scale factor. Measured velocity (u) is multiplied by this value to give mean velocity in the channel. Similar relationships can be obtained for different channel types.

Gonzales, et al (1996) compares theoretical profiles with data collected with an ADCP. They estimate an exponential value of m=1/6, which agrees with Manning's equation. For modeling in the Argonaut firmware, we use m=1/6.

Theoretical Flow – Argonaut-SL

The Argonaut-SL measures velocity at one depth within the water column. Comparing that depth with the total channel depth allows us to estimate the relationship between the measured velocity (u) and the mean channel velocity (V). Using m=1/6 in Equation 4, we get:

Equation 5 $$\frac{V}{u} = \frac{6}{7}\left(\frac{h}{y}\right)^{\frac{1}{6}}$$

This relationship is used for any channel type (irregular, trapezoidal, or pipe), although in reality an SL is not appropriate for installation in a pipe. For practical installations, the measured SL velocity is scaled by values ranging from 0.86 (measurement location near the surface) to 1.20 (measurement location near the bottom) to give the mean channel velocity (see Figure 1 for a plot of the velocity distribution). Mean channel velocity is multiplied by channel area (based on water level and channel geometry) to get flow.

Theoretical Flow – Argonaut-SW

The Argonaut-SW is mounted on the bottom, looking up, and vertically integrates velocity over some portion of the water column (from the lowest possible point to the surface). The starting point is determined by a combination of the blanking distance, pulse length, and the mounting height of the instrument. For standard operating parameters, the first point is about 15 cm above the transducer head, and 20-25 cm above the bottom of the channel.

To relate SW velocity to mean channel velocity, we predict how the integrated velocity measured by the SW compares with the mean channel velocity.

Open Channels

For open channels (regular or irregular shapes), we start with Equation 4. We look only at the vertical velocity distribution and ignore the effects of channel walls. We integrate over the portion of the water column measured by the SW to get a relationship of the measured velocity to the mean channel velocity.

Equation 6
$$\frac{V}{u_{meas}} = \frac{\int_{y_0}^{h} \frac{1}{m+1}\left(\frac{h}{y}\right)^{m} dy}{h - y_0}$$

This can be reduced to the following.

Equation 7
$$\frac{V}{u_{meas}} = \frac{1-\left(\frac{y_0}{h}\right)^{1-m}}{(m+1)(1-m)\left(1-\frac{y_0}{h}\right)}$$

Where:

u_{meas} = Argonaut-SW measured velocity
y_0 = starting point for SW integrated velocity cell (from bottom of channel)

Substituting m=1/6 in the above gives the following.

Equation 8
$$\frac{V}{u_{meas}} = \frac{36\left(1-\left(\frac{y_0}{h}\right)^{5/6}\right)}{35\left(1-\frac{y_0}{h}\right)}$$

For practical situations, the SW measured velocity is scaled by values ranging from 0.88 (30 cm deep, top 30% of the water column is measured) to 0.98 (400 cm deep, over 90% of water column is measured) to give the mean velocity in the channel. Mean velocity is multiplied by channel area (based on water level and channel geometry) to give flow. Plots of the velocity coefficients, both single point (Equation 5) and integrated (Equation 8) are shown in Figure 1.

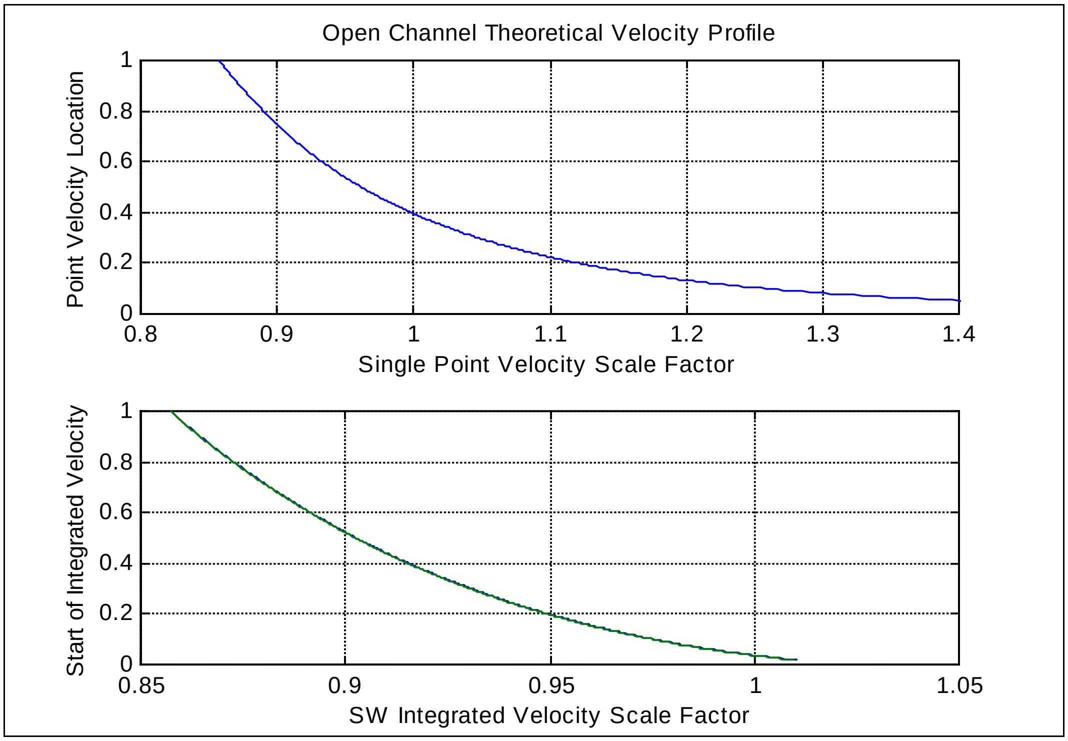

Figure 1 - Theoretical Velocity Profiles for Open Channel Flow

Flow in Pipes - Full

For flow in full pipes, we develop an equation analogous to Equation 4. We integrate the power law (Equation 1) over a pipe cross section to relate mean pipe velocity to shear velocity. The distance to the nearest boundary (y) is the radial distance to the channel wall.

Equation 9 $$\frac{V}{u*} = \frac{2a}{(m+1)(m+2)}\left(\frac{r_0}{y'}\right)^m$$

Where

r_0 = pipe radius

We combine Equation 1 and Equation 9 to get the following relationship.

Equation 10 $$\frac{V}{u} = \frac{2}{(m+1)(m+2)}\left(\frac{r_0}{y}\right)^m$$

The SW measures integrated velocity from some point within the water column (typically 20-25 cm off the bottom) to the top of the pipe. We consider two cases to develop a relationship between integrated and mean velocity: start of measurement volume above mid point of the pipe, and start of measurement volume below the mid point of the pipe. If the measurement volume starts above the mid-point of the pipe:

Equation 11 $$\frac{V}{u_{meas}} = \frac{\int_0^{y_0} \frac{2}{(m+1)(m+2)} \left(\frac{r_0}{y}\right)^m dy}{y_0} = \frac{2}{(m+1)(m+2)(1-m)} \left(\frac{r_0}{y_0}\right)^m$$

Substituting m=1/6:

Equation 12 $$\frac{V}{u_{meas}} = 0.9495 \left(\frac{r_0}{y_0}\right)^{\frac{1}{6}}$$

If the measurement volume starts below the mid point of the pipe, we combine two integrals (one for the top half of the pipe, one for the fraction of the bottom half that is measured):

Equation 13 $$\frac{V}{u_{meas}} = \frac{\int_0^{r_0} \frac{2}{(m+1)(m+2)} \left(\frac{r_0}{y}\right)^m dy + \int_{y_0}^{r_0} \frac{2}{(m+1)(m+2)} \left(\frac{r_0}{y}\right)^m dy}{2r_0 - y_0}$$

$$\frac{V}{u_{meas}} = \frac{2\left(2 - \left(\frac{y_0}{r_0}\right)^{1-m}\right)}{(m+1)(m+2)(1-m)\left(2 - \frac{y_0}{r_0}\right)}$$

Substituting m=1/6:

Equation 14 $$\frac{V}{u_{meas}} = 0.9495 \frac{\left(2 - \left(\frac{y_0}{r_0}\right)^{\frac{5}{6}}\right)}{\left(2 - \frac{y_0}{r_0}\right)}$$

The SW measured velocity is scaled by values ranging from 1.03 (30 cm diameter, top 30% of the water column is measured) to 0.92 (200 cm diameter, 90% of water column is measured) to give the mean velocity in the pipe. Plots of the velocity scale factors (both for single point and integrated velocity) are shown in Figure 2.

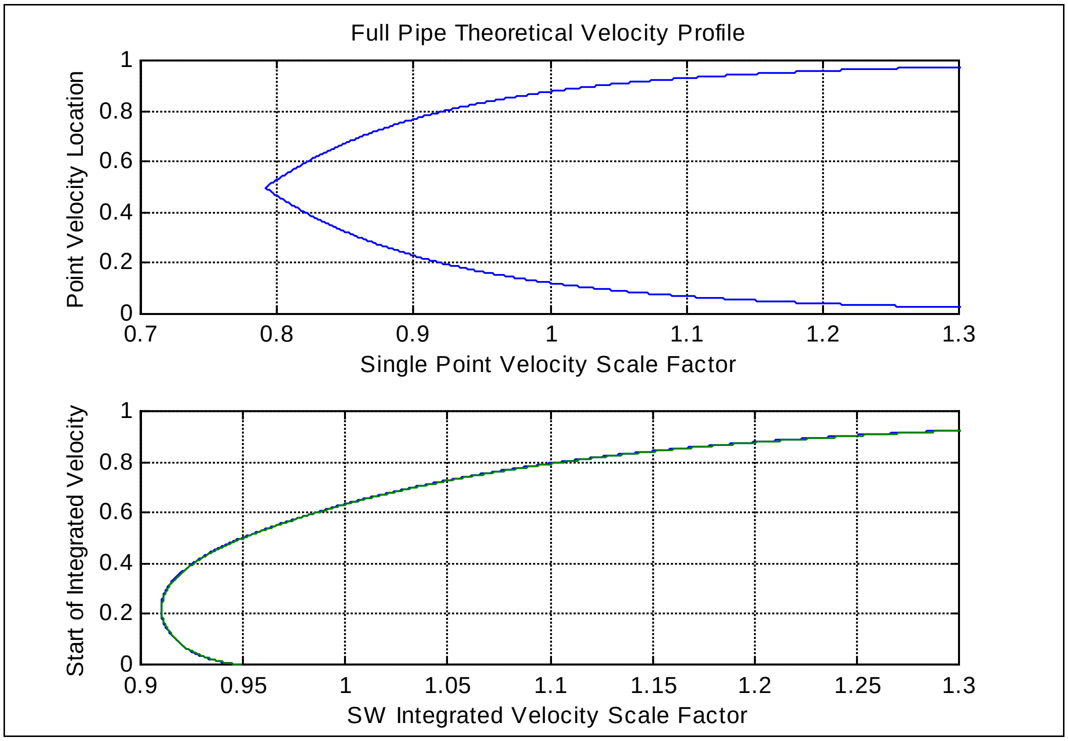

Figure 2 – Theoretical Velocity Profiles for Full Pipe

Flow in Pipes – Partially Full

Flow in partially full pipes presents a difficult problem because of the addition of another variable (the fraction of the pipe that is filled), and the complexity of determining the distance to nearest boundary. To solve this problem, we use the following relation derived starting with Equation 1.

Equation 15
$$V = \frac{\iint u \cdot dA}{\iint dA} = \frac{\iint au^* \left(\frac{y}{y'}\right)^m dA}{\iint dA}$$

Combining equation 15 with equation 1 to eliminate u*, we get the following.

Equation 16
$$\frac{V}{u} = \left(\frac{1}{y}\right)^m \frac{\iint y^m dA}{\iint dA}$$

Where:

$\iint$ = integration over the wetted area of the pipe

y = distance to the nearest boundary

For any given condition (fraction of the pipe that is full), the ratio of the two double integrals is constant, reducing equation 16 to the following.

Equation 17 $$\frac{V}{u} = A\left(\frac{1}{y}\right)^m$$

We think of the ratio (V/u) as a velocity scale factor; we multiple measured velocity (at a given point in the pipe) by this scale factor to determine mean velocity in the channel.

To determine the relationship between the SW measured velocity and mean velocity in the pipe, we integrate Equation 17 over the portion of the pipe measured by the SW. SW velocity measurements are assumed to be along the centerline of the pipe. The integration is fairly simple if the pipe is less than half full. The integration become much more difficult for a pipe that is more than half full (but not completely full) due to the complexity of determining the closest distance to the wetted wall of the pipe. Because of this complexity, the integration was done numerically as described below.

The numerical approximation was run at different water levels.

- The pipe was divided into a number of small elements (~1 million).
- The ratio of the integrals in Equation 16 was numerically estimated for each water level.
- A profile of the velocity scale factor (V/u) was created along the centerline of the pipe.
- Figure 3 shows a plot of the this scale factor for pipes ranging from 10 to 100% full (in 10% increments). The profile is along the centerline of the pipe.

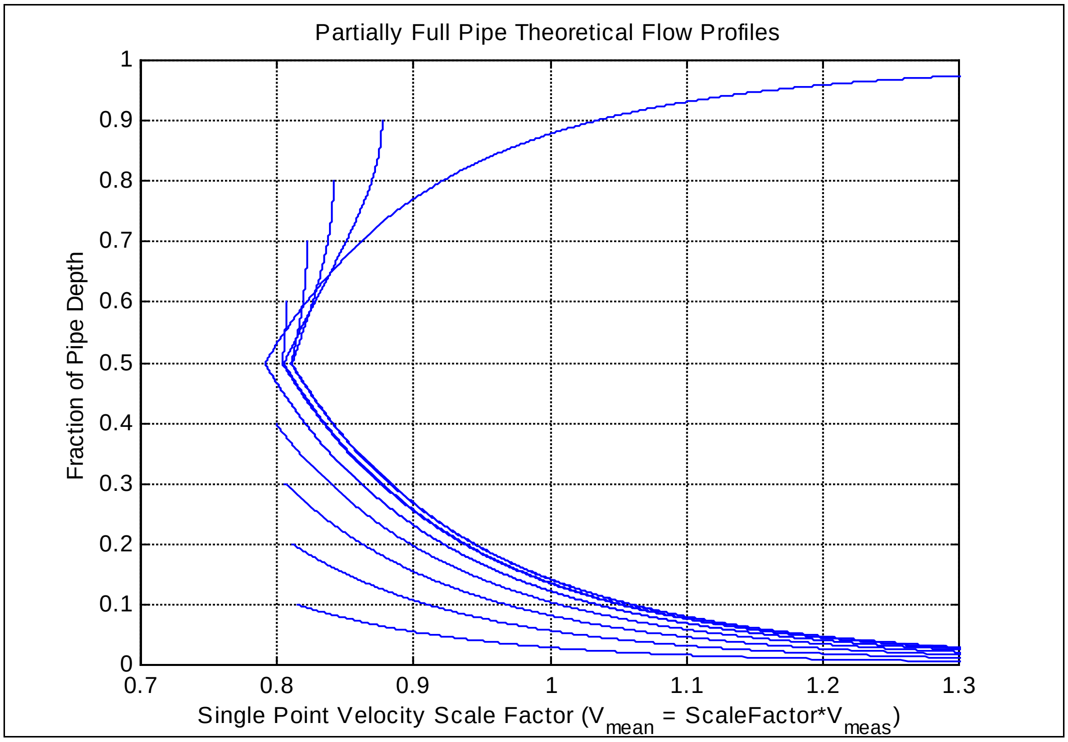

Figure 3 – Theoretical Velocity Profiles for a Partially Full Pipe

To relate the SW measured velocity to mean velocity in the pipe, we need to account for the portion of the water column measured by the SW.

- The SW integrates velocity over a portion of the water column, starting some distance from the bottom of the pipe and ending at the water surface.
- The profile of the single point velocity scale factor was integrated from the surface down. The ending point of this integration corresponds to the first point in the integrated velocity measurement made by the SW.
- We perform this integration for any measured fraction of the water column, since the portion of the water column measured by the SW will vary depending on water depth, sensor installation, and system operating parameters.
- All calculations are normalized, looking at the fraction of the pipe that is full and the faction of the water column that is measured. They can be applied to a pipe of any dimension.
- Figure 4 shows the results of this integration for pipes ranging from 10 to 100% full. Water depth is plotted as fraction of the total water depth, rather than as a fraction of the pipe depth.
- This graph shows the scale factor by which SW velocity data is multiplied to give mean velocity in the pipe. To use this graph:
    - Select the curve corresponding to the fraction of the pipe that is full.
    - On the vertical axis, pick the fraction of the water depth measured by the SW.
    - The value shown for that curve at that level gives the SW velocity scale factor. Multiply SW velocity by this value to get mean velocity in the pipe.

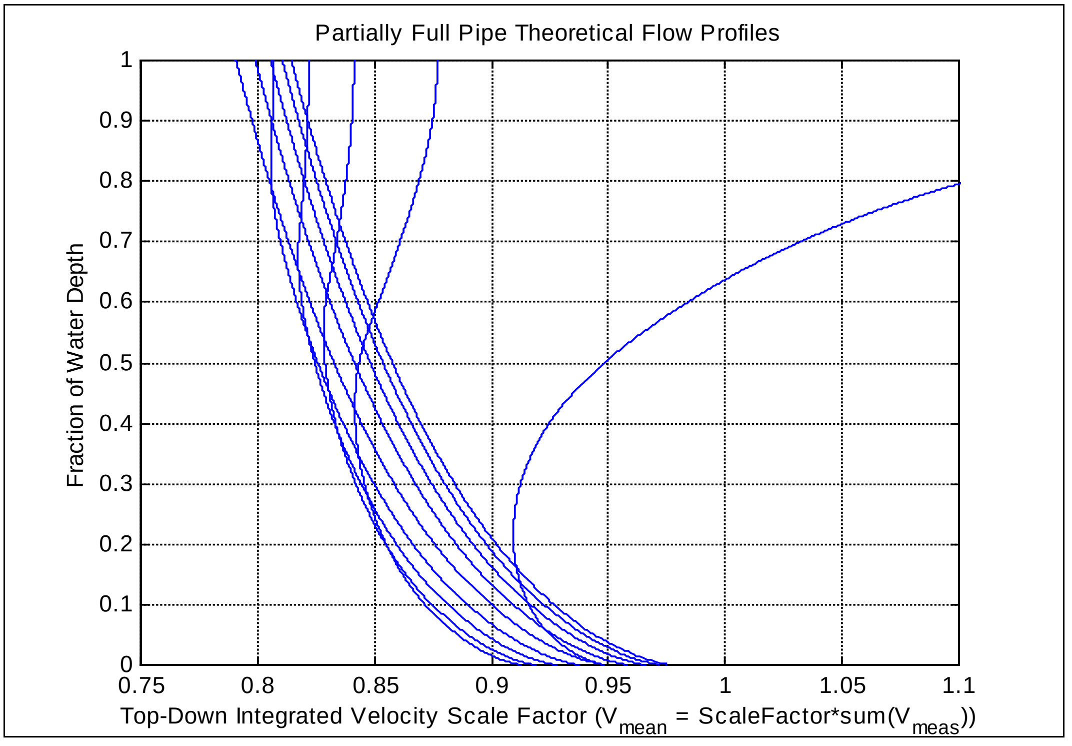

Figure 4 – Theoretical Integrated Velocity Profiles for a Partially Full Pipe

Results for a full pipe were compared with the symbolic solution shown earlier. The level of agreement depended on the number of points used for the calculations; for the final calculations (~1 million points) the agreement was better than 0.1%.

Some notes about the SW velocity scale factor.

- The minimum practical value is about 0.81 (50 cm diameter pipe flowing 60% full, top 30% of water column is measured).
- The maximum practical value is about 1.03 (30 cm diameter pipe flowing full, top 30% of water column is measured).
- The largest changes occur in nearly full pipes. The difference between a pipe flowing 90% and 100% full can be more change the scale factor by more than 10%. The changes are exaggerated in small pipes, where a smaller portion of the water column is measured.

Within the Argonaut-SW, we implement the results as follows.

- A look up table was created using the numerical simulations.
- Simulations were done for pipes ranging from 5 to 100% full, in 5% increments.
- For each condition we will consider measurements that cover from 5 to 100% of the velocity profile in 5% increments. In practical situations, the SW should never measure less than 30% nor more than 95% of the water column.
- Based on the actual conditions (fraction of pipe that is full, and fraction of water column that is measured) we interpolate from the nearest points in the look up table to estimate the velocity scale factor.
- Measured velocity is multiplied by the scale factor, and then by wetted area, to give flow in the pipe.

Assumptions and Limitations

These theoretical models use some basic assumptions that should be kept in mind when evaluating their usefulness.

Power law

- We assume the flow follows a power law velocity distribution with m=1/6.
- This effectively makes assumptions about bottom roughness and boundary conditions.

Open channel

- For open channel flow, we look only at the vertical velocity distribution.
- We do not account for the influence of channel walls. This is at least partially supported for concrete channels, as the bottom will typically be covered in sediment giving higher flow resistance compared to the relatively smooth channel walls.
- Our modeling assumes constant depth across the channel.

Pipe flow

- We assume uniform roughness around the diameter of the pipe.

References

Chen, C.L. “Unified theory on power laws for flow resistance,” Journal of Hydraulic Engineering, ASCE, Vol. 117, No. 3, March 1991, pp. 371-389.

Gonzalez, J.A., Melching, C.S., Oberg, K.A., "Analysis of open-channel velocity measurements collected with an acoustic Doppler current profiler," Rivertech 96 Proceedings, September 1996.